

## **causal-bijection\_functionality\_wrt\_iff<sup>11,40</sup>**

$\forall es:ES, T, T':\text{Type}, R:(T \rightarrow T' \rightarrow \mathbb{P}), P_1, P_2, Q_1, Q_2:(E \rightarrow \mathbb{P}), f:(\{e:E| P_1(e)\} \rightarrow \{e:E| Q_1(e)\})$ .  
 $(\forall e:E. P_1(e) \Rightarrow (\text{valtype}(e) \subseteq_r T))$   
 $\Rightarrow (\forall e:E. Q_1(e) \Rightarrow (\text{valtype}(e) \subseteq_r T'))$   
 $\Rightarrow (\forall e:E. P_1(e) \iff P_2(e))$   
 $\Rightarrow (\forall e:E. Q_1(e) \iff Q_2(e))$   
 $\Rightarrow ((e.P_1(e) \rightarrow e.f(e) \rightarrow e.Q_1(e)) \text{ with } R \iff (e.P_2(e) \rightarrow e.f(e) \rightarrow e.Q_2(e)) \text{ with } R)$